Addendum

Search for Charm

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I. DISCOVERY OF NARROW RESONANCES AT 3.1 AND 3.7 GeV

Since this article was written there have been several dramatic observations of narrow resonances in e^+e^- and other channels.

In the reaction $p + Be \rightarrow e^+ + e^- + \text{anything}$, Aubert et al., (1974a) have observed a sharp enhancement at $M(e^+e^-) = 3.1$ GeV. The experiment was performed at the Brookhaven 30 GeV alternating-gradient synchrotron.

In independent experiments at the same time, a SLAC-LBL group has observed sharp resonant peaks around 3.1 GeV in the colliding beam processes: $e^+ + e^- \rightarrow$ hadrons, $e^+ + e^- \rightarrow \mu^+ + \mu^-$ and $e^+ + e^-$, at SPEAR (Augustin et al., 1974). The mass and width as measured at SPEAR are

$$m = 3.105 \pm 0.003 \text{ GeV}, \tag{A.1}$$

$$\Gamma \leq 1.9 \text{ MeV}. \tag{A.2}$$

Later observations at SPEAR of the interference between the one-photon and resonant contributions to $e^+ + e^- \rightarrow \mu^+ + \mu^-$ suggest that the 3100 MeV resonance has $J^{PC} = 1^{--}$ (B. Richter, private communication).

The observed width (A.2) is a convolution of the actual width with the beam energy resolution and the radiative correction due to soft photon emission (Bonneau and Martin, 1971; Jackson, 1974a; Yennie, 1974). By integrating over the resonant part of $\sigma(e^+ + e^- \rightarrow \text{hadrons})$ one obtains (for example, Jackson, 1974a).

$$\frac{\Gamma(3.1\ \text{GeV} \to e^+ + e^-)\,\Gamma(3.1\ \text{GeV} \to \text{hadrons})}{\Gamma_{\text{total}}(3.1\ \text{GeV})} \simeq 5\ \text{keV}. \qquad (A.3)$$

The ratio of the integrated resonant cross sections for $e^+ + e^- \to \mu^+ + \mu^-$ and $\to$ hadrons give

$$\frac{\Gamma(3.1\ \text{GeV} \to \mu^+ + \mu^-)}{\Gamma(3.1\ \text{GeV} \to \text{hadrons})} \simeq 7\% \quad . \qquad (A.4)$$

Thus, if we assume that there are no purely neutral decays, and $\Gamma(e^+e^-) = \Gamma(\mu^+\mu^-)$ for which there seems to be some support, i.e., $\Gamma_{\text{total}} = \Gamma(\text{hadrons}) + 2\Gamma(\mu^+\mu^-)$, then we have

$$\Gamma(3.1\ \text{GeV} \to e^+ + e^-) \simeq 6\ \text{keV},$$
$$\Gamma_{\text{total}}(3.1\ \text{GeV}) \simeq 92\ \text{keV}. \qquad (A.5)$$

The resonance at 3.1 GeV has also been observed in colliding experiments at ADONE (Frascati) by Bacci et al., (1974) and at DORIS (Hamburg) by Braunschweig et al., (1974).

Within 10 days of its first discovery, the SLAC-LBL group at SPEAR found another resonance in the process $e^+ + e^- \to$ hadrons (Abrams et al., 1974). The second resonance is quoted to have

$$m = 3.695 \pm 0.004\ \text{GeV}, \qquad (A.6)$$

$$\Gamma \leq 2.7\ \text{MeV}\ . \qquad (A.7)$$

By the integration method used to deduce (A.3) and (A.4), one obtains

$$\frac{\Gamma(3.7\ \mathrm{GeV} \to \mu^+ + \mu^-)}{\Gamma(3.7\ \mathrm{GeV} \to \mathrm{hadrons})} < 2\% ,$$

$$\Gamma(3.7\ \mathrm{GeV} \to \mu^+ + \mu^-) \simeq 2.4\ \mathrm{keV}, \tag{A.8}$$

and

$$\Gamma_{total}(3.7\ \mathrm{GeV}) > 125\ \mathrm{keV}.$$

This resonance was not seen in the reaction $p + Be \to \mu^+ + \mu^- + \text{anything}$ at Brookhaven (Aubert, 1974b).

The decays that have been identified so far include:

$$3100 \to e^+e^-,\ \mu^+\mu^- \tag{A.9}$$

$$\to 2\pi^+ 2\pi^- (\pi^0) \tag{A.10}$$

$$\to \omega\pi^+\pi^- \tag{A.11}$$

$$\left.\begin{array}{l} \to p\bar{p} \\ \to K^+K^- \end{array}\right\} \text{tentative} \tag{A.12}$$

$$3700 \to 3100 + \pi^+ + \pi^- . \tag{A.14}$$

(B. Richter, private communication). The decays (A.11) and (A.14) indicate that the G-parity of both resonances is odd (unless the decay (A.11) is occurring electromagnetically). A very little $3100 \to 2\pi^+ 2\pi^-$ is seen, further strengthening the odd G-parity assignment. In the process (A.14), the $\pi^+\pi^-$ system seems to be in the $I = 0$, $J = 0$ state; it is very similar to ρ' (1600) $\to \rho + \pi^+ + \pi^-$, and is suppressed compared to $\rho' \to \rho + \pi^+ + \pi^-$ to the same extent as the width of the 3.1 GeV resonance is compared to the width of ρ. It is likely that the decay (A.14) is about

30% of the visible decays of the 3.7 GeV resonance (Jackson, 1974b). In fact, observations are in agreement with the decay scheme: $1^- \rightarrow 1^- + 0^+(\pi\pi)$.

II. THE NEW STATES AS ($c\bar{c}$) BOUND STATES

Many interpretations have been advanced for the new resonances. It is not our purpose here to discuss them all. We shall concentrate on the possibility that the new states correspond to:

3100 MeV: ϕ_c, a $^3S_1$ $c\bar{c}$ state, partner of ρ, ω, ϕ
(see Table III and Secs. 3.3, 4.4)

3700 MeV: ϕ_c', a radially excited $^3S_1$ $c\bar{c}$ state,
possible partner of ρ' (1600).

If this is the case, the new particles should be accompanied by a host of others with <u>nonzero charm</u> (the present ones have charm = 0 since they are made of $c\bar{c}$). (See also Harari, 1974; CERN Meson Workshop, 1974). The charmed particles should have properties even more dramatic than those of the above states. The lowest ones should decay only weakly and have lifetimes of order 10^{-13} sec. Their masses should lie between 2 and 2.5 GeV. This is a considerable narrowing of the range of our earlier estimates. It is made possible by the fact that the new particles have set the mass scale in a charm model. If these new particles are not associated with charm, the scale reverts to that we have mentioned previously.

The particle at 3100 MeV has been called J by the MIT-Brookhaven group (Aubert et al., 1974a) and ψ by the SLAC-LBL group (Augustin et al., 1974). We shall call it ϕ_c for the remainder of this discussion while realizing that another name may well have been chosen by the time this article is in print.

The narrow widths of the ϕ_c and the ϕ_c' can be understood qualitatively if they lie below threshold for production of a pair of charmed hadrons. The hadronic vertices of "normal" strength always appear to involve connected quark graphs. Processes which do not involve such graphs, such as $\phi \rightarrow$ (nonstrange hadrons) and presumably $\phi_c \rightarrow$ (charmless hadrons), are subject to considerable suppression.[2] In the case of $\phi \rightarrow \rho\pi$, this suppression is a factor of at least 10^2. Nonetheless, one must assume that the suppression for ϕ_c is about 20 times stronger than for the ϕ, as a comparison of (A.4) with our prediction (4.16) shows. This is either a serious problem or an important result, depending on one's point of view. In any case, understanding the so-called Zweig's rule is a serious challenge to theorists. For example, the tremendously small width of ϕ_c may be evidence that the strong interactions are getting weaker at high energies. (See Appelquist and Politzer, 1975; De Rújula and Glashow, 1975). In "asymptotically free" theories the mixing of ϕ with nonstrange states and of ϕ_c with uncharmed states is governed by the annihilation of the quark-antiquark pair ($s\bar{s}$ or $c\bar{c}$) into three gluons (the minimum number consistent with color and charge-parity conservation). The rate for this process is proportional to the twelfth power of the running coupling constant, which gets weaker for high masses and short distances, if the system is "Coulombic". By suitable rescaling of this coupling constant using the renormalization group, one can reduce the prediction (4.16) to agree with (A.4). Since there is some question as to how many powers of the running coupling constant should actually be scaled in such an exercise, we shall not quote details, which may be found in the papers just mentioned.

The leptonic decay widths of ϕ_c were estimated in Sec. 4.4, by assuming that the photon-ϕ_c coupling behaved as $e\, m_{\phi_c}^{2}/\gamma_{\phi_c}$ and applying the quark model to the ratio

$$\gamma_{\phi_c}^{-1} : \gamma_{\phi}^{-1} = 2:-1 . \tag{A.15}$$

In this manner, for $m_{\phi_c} = 3.1$ GeV, we estimated

$$\begin{aligned} \Gamma(\phi_c \to \mu^+\mu^-) &= \Gamma(\phi_c \to e^+e^-) \\ &= 4\, \frac{m_{\phi_c}}{m_{\phi}}\, \Gamma(\phi \to e^+e^-) \\ &= 12\,(3.2 \times 10^{-4})\,(4.2\ \text{MeV}) \\ &\simeq 16.4\ \text{keV}. \end{aligned} \tag{A.16}$$

This is to be compared with the experimental value in (A.5):

$$\Gamma(\phi_c \to \mu^+\mu^-) = \Gamma(\phi_c \to e^+e^-)$$
$$\simeq 6 \text{ keV}. \tag{A.17}$$

It has been argued that one should not apply the quark model to the quantities γ_V^{-1} (as in Eq. (A.15)) but rather to the combinations m_V/γ_V. These arguments date from 1967 (Das et al., 1967b) and rely on the use of the first spectral function sum rule of Weinberg (1967b) and Das et al., (1967a), for asymptotic SU(3). A trivial extension of this sum rule to asymptotic U(4), continuing to use single-particle saturation, would predict[4]

$$m_{\phi_c}/\gamma_{\phi_c} : m_\phi/\gamma_\phi = 2:-1 , \tag{A.18}$$

and hence (comparing with Eq. (A.16))

$$\Gamma(\phi_c \to \mu^+\mu^-) = \Gamma(\phi_c \to e^+e^-)$$
$$= \left(\frac{m_\phi}{m_{\phi_c}}\right)^2 (16.4 \text{ keV})$$
$$\simeq 1.8 \text{ keV}. \tag{A.19}$$

The two predictions bracket the experimental result. Symmetry-breaking effects of this sort will not be calculable without precise dynamical models for the $c\bar{c}$ system.[5] (See also Yennie, 1974.)

One can make a similar range of predictions for the leptonic decays of the ϕ_c' by assuming that it is the charmed analogue of the

ρ' (1600) and that both states are $^3S_1$ radial excitations. Instead, we shall use the ratio of (A.17) to (A.8) to extract the coupling ratio: in the "naive" model,

$$\frac{\gamma_{\phi_c'}^{2}}{\gamma_{\phi_c}^{2}} = \frac{\Gamma(\phi_c \to \text{leptons})}{\Gamma(\phi_c' \to \text{leptons})} \; \frac{m_{\phi_c'}}{m_{\phi_c}} \simeq 2. \tag{A.20}$$

This is to be compared to the ratio obtained from ρ' photoproduction:

$$4 < \frac{\gamma_{\rho'}^{2}}{\gamma_{\rho}^{2}} < 8 \; . \tag{A.21}$$

(For a review, see Moffeit, (1973)). The qualitative agreement suggests that it is reasonable to take ϕ_c' as a radial excitation of the ϕ_c.

As long as the ϕ_c' is below charm-anticharm threshold, it will remain narrow. (This is an important difference between the ϕ_c' amd the ρ', which has a number of open channels into which to decay and is consequently very broad.) <u>The existence of a narrow ϕ_c' at 3700 MeV means that the lowest charmed particle must lie above 1850 MeV in mass.</u> In a deep nonrelativistic square-well potential, for example, the second radial excitation would lie $1\frac{2}{3}$ times as high above ϕ_c' as ϕ_c' lies above ϕ_c, i.e., at 4700 MeV. In almost any other potential one can imagine, a second radial excitation would lie no higher above ϕ_c' than ϕ_c' lies above ϕ_c. The total cross section has a peak about 10 nb high and 200 MeV wide at 4.1 GeV (Augustin, et al., 1975). This might be a second radial excitation of ϕ_c, above threshold. In any case, the charm-anticharm threshold probably lies somewhere between 3.7 and 4.7 GeV. The lowest-mass charmed hadron probably lies between 1850 and 2350 MeV, unless the potential between c and $\bar{c}$ is of a very unusual sort or unless the higher discrete states are considerably broader than those at 3.1 and 3.7 GeV.

III. OTHER ($c\bar{c}$) STATES AND THEIR DECAYS

The Coulombic $c\bar{c}$ system has been referred to as "charmonium" by Appelquist and Politzer (1975) in analogy with positronium. In this picture, the ϕ_c and ϕ_c' are states of "orthocharmonium" ($^3S_1$); they are expected to have hyperfine "paracharmonium" ($^1S_0$) partners, which we shall call η_c and η_c'. These states have $J^{PC} = 0^{-+}$.

Quite independently of the Coulombic nature of the force acting between c and $\bar{c}$, it is expected that the $c\bar{c}$ system has a structure of levels below charm-anticharm threshold associated with radial and orbital excitations. All such levels are expected to be narrow. Because of this, they will provide the first test of detailed quark-antiquark potentials (see for example, Appelquist et al., 1975; Eichten et al, 1975; Callan et al., 1975; Schnitzer, 1974).

These states other than the ones with $J^{PC} = 1^{--}$ cannot be produced directly in e^+e^- interactions via one photon intermediate state. However, some of them could be more easily produced in hadronic reactions such as that utilized by Aubert et al., (1974). This is because some of them can communicate with the charmless hadron world via two-gluon exchange, whereas $(c\bar{c})$ states of odd charge conjugation parity communicate via three-gluon exchange (see De Rújula and Glashow, 1975). If the gluon-quark coupling constant is weak, as one might infer from the narrowness of ϕ_c, this can give a considerable advantage to η_c production.[6] Note that this observation is independent of the Coulombic nature of $(c\bar{c})$ states.

Further indications of the relative strengths of two-gluon and three-gluon processes may be obtained by comparing the mixing of the "ordinary" vector and pseudoscalar mesons. The vector mesons are nearly "ideally" mixed ($\omega \simeq (u\bar{u} + d\bar{d})/\sqrt{2}$; $\phi \simeq s\bar{s}$) while the pseudoscalars are nearly pure members of SU(3) multiplets ($\eta \simeq (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$; $\eta' \simeq (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$). This can be understood if, in addition to a quark mass term, the mass operator contains an additional term associated with the transition $q\bar{q} \rightarrow$ (2 or 3 gluons) $\rightarrow q'\bar{q}'$ in the s-channel. The pseudoscalar mesons are mixed via 2-gluon exchange (strongly) and the vector mesons via 3-gluon exchange (weakly).

If this picture is extended to the charm model, one again expects the 0^- states to be more strongly mixed than the 1^- states. However, in the text (Sec. 3.2) we have proposed a solution in which the $\eta'(958)$ is an SU(4) singlet, and thus spends 1/4 of the time as a $c\bar{c}$ pair. This solution now seems unlikely, as it would entail a width for $\phi_c \rightarrow \eta'\gamma$

exceeding the total width of the ϕ_c. This process is related to $\omega \to \pi^0\gamma$ by a Clebsch-Gordon coefficient: neglecting kinematic factors,

$$\Gamma(\phi_c \to c\bar{c}(0^-) + \gamma)/\Gamma(\omega \to \pi^0\gamma) = 16/9\ , \qquad \text{(A. 22)}$$

where Γ is the width with kinematic factors divided out. If one adopts the prescription of Gilman and Karliner (1974), $\Gamma \sim \tilde{\Gamma}(p_\gamma)^3$, one finds that the η' must be spending less than 10^{-3} of the time as $c\bar{c}$. A similar conclusion follows from vector dominance with the coupling prescription (A. 15). If $\Gamma \sim \tilde{\Gamma} m_V$, one still finds that the η' cannot be more than one or two percent $c\bar{c}$. Similar arguments apply to the η. Consequently, there must exist another 0^- state which is dominantly $c\bar{c}$, though probably considerably less pure than the ϕ_c. It is this state to which we shall refer as η_c.

The choice $\eta_c \simeq c\bar{c}$ was dismissed in our article as giving a poor fit to pseudoscalar masses. Recently this fit was re-examined by Lee and Quigg (1974). Two solutions were found, based on a value of $R \equiv (m_c - m_u)/(m_s - m_u)$ consistent with the ϕ_c mass:

$$\eta \simeq 0.8\left(\frac{u\bar{u} + d\bar{d}}{\sqrt{2}}\right) - 0.6\ s\bar{s},\ m_\eta = 508\ \text{MeV},$$

$$\eta' \simeq 0.6\left(\frac{u\bar{u} + d\bar{d}}{\sqrt{2}}\right) + 0.8\ s\bar{s},\quad m_{\eta'} = 969\ \text{MeV}, \qquad \text{(A. 23)}$$

$$\eta_c \simeq 1.00\ c\bar{c},\qquad m_{\eta_c} = 3122\ \text{MeV},$$

with $c\bar{c}$ admixtures in η and η' of less than a percent, and

$$\eta \simeq 0.66 \left(\frac{u\bar{u} + d\bar{d}}{2}\right) - 0.75 \quad s\bar{s}, \; m_\eta = 551 \text{ MeV},$$

$$E \simeq 0.75 \left(\frac{u\bar{u} + d\bar{d}}{2}\right) + 0.66 \quad s\bar{s}, \; m_E = 1398 \text{ MeV},$$

$$\eta_c = 1.000\, c\bar{c}, \; m_{\eta_c} = 3066 \text{ MeV}. \tag{A. 24}$$

with $c\bar{c}$ admixtures for η and E less than a percent.
The fit (A. 23) is not particularly close to the η mass, but with such large mixing effects perhaps one should not expect better. The fit (A. 24) requires that one identify the E(1420) with the ninth member of the 0^- nonet. In either case, however, the η_c is relatively pure $c\bar{c}$ and is fairly close to the ϕ_c.

The Coulombic estimate (Appelquist et al., 1975) for the hyperfine splitting between ϕ_c and η_c gives

$$m_{\phi_c} - m_{\eta_c} = \frac{1}{6}\left(\frac{9}{\alpha^2} \frac{\Gamma(\phi_c \to e^+e^-)}{m_{\phi_c}}\right)^{4/3} m_{\phi_c}$$

$$\simeq 80 \sim 90 \text{ MeV}, \tag{A. 25}$$

with a smaller splitting expected between ϕ_c' and η_c'. A similar estimate is obtained more phenomenologically by assuming $m_{\phi_c}^2 - m_{\eta_c}^2 \simeq m_\rho^2 - m_\pi^2$. Several older experiments have seen bumps between 3.0 and 3.1 GeV (French, 1968; Alexander et al., 1970; Braun et al., 1971) and the MIT-Brookhaven group may also have observed the effect. In all of these experiments the $p\bar{p}$ channel plays a crucial role.

The experiments mentioned by French (1968) involve $p\bar{p} \to$ pions at 5.7 GeV/c. Among the numerous peaks mentioned, there is one in the $(4\pi)^0$ system at 3.08 GeV. In a similar experiment at 7.0 GeV/c, Alexander et al. (1970) see peaks in the $(6\pi)^0$ channel at 3.035 and 3.4 GeV. It is amusing that I = 0, G = + are precisely the quantum numbers one expects for the $c\bar{c}$ states that can be produced via two-gluon exchange, and hence the most likely quantum numbers for $c\bar{c}$ states coupled to charmless hadrons. In order for the above states to have anything to do with charm, they must be much narrower than quoted in the literature.

Braun et al. (1971) see a peak in the $p\bar{p}$ distribution at 3.05 GeV in the reaction $\bar{p}d \to p_s p\bar{p}\pi^-$ at 5.5 GeV/c. Its statistical significance is marginal.

The $p\bar{p}$ channel is actually an ideal one for the study of 0^- $c\bar{c}$ states, independently of the above experiments. It is the most readily accessible two-body channel; another is $\gamma\gamma$, which we shall discuss shortly, and still another is $\Lambda\bar{\Lambda}$. Since one expects the $c\bar{c}$ state to be a unitary singlet, the $\Lambda\bar{\Lambda}$ decay rate should equal the $p\bar{p}$ rate, modulo slowly varying kinematical factors: $\Gamma_{\Lambda\bar{\Lambda}}/\Gamma_{p\bar{p}} = (m_{\eta_c}^2 - 4m_\Lambda^2)^{1/2}/(m_{\eta_c}^2 - 4m_p^2)^{1/2} \simeq 0.87$.

Several of our colleagues (for example, R. Cahn) have suggested that the bump in total $p\bar{p}$ cross sections around E_{CM} = 1.93 GeV (Carroll et al., 1974) might be related to the η_c. If this is the case, it cannot be the hyperfine partner of the 3100 MeV resonance. Aside from the estimate based on (A.25), one can place a lower bound on the mass of an $\eta_c \sim c\bar{c}$ by using (A. 22). (We have argued that η and η' have very little $c\bar{c}$, so that there must exist such a state). The results are shown in Table A. 1.

Table A. 1

Predicted widths for $\phi_c \to \eta_c \gamma$, keV

m_{η_c} (GeV)	$\Gamma \sim \tilde{\Gamma}(p_\gamma)^3$	$\Gamma \sim \tilde{\Gamma}(p_\gamma{}^3/m_V{}^2)$
2.7	1600	$\underline{100}$
2.95	$\underline{100}$	6
3.05	5	0.3

The value 100 keV, underlined in the table, corresponds to the total width of the ϕ_c. The two barrier factors correspond to the prescription of Gilman and Karliner (1974) and to the assumption of vector dominance with couplings obeying (A. 18), respectively.[7] [The width for $\phi_c \to \eta_c\gamma$ predicted by Appelquist et al. (1974) is even smaller than any of the above values: about 0. 03 keV.]

On the basis of Table A. 1, the η_c must lie above 2.7 GeV, or the ϕ_c would be too wide.

Let us assume the η_c has a mass of 3. 05 GeV and estimate its $\gamma\gamma$ width. Scaling the π^0 width up as m^3 [this follows in a treatment of the axial-vector anomaly, or in a vector dominance model with "naive" couplings as in (A. 15); the m^3 ansatz works well for π^0, $\eta \to 2\gamma$ decays] and using $\Gamma(\eta_c \to \gamma\gamma) = (32/9)\,\Gamma(\pi^0 \to \gamma\gamma)$, one obtains $\Gamma(\eta_c \to \gamma\gamma) \simeq 300$ keV. The experimental photon-ϕ_c coupling, as deduced from a comparison of (A.16) and (A.17), is probably about a factor of 1.7 to 2 smaller than the "naive" value, leading to the modified estimate based on vector dominance of

$$\Gamma(\eta_c \to \gamma\gamma) \simeq 20 \text{ to } 40 \text{ keV}. \tag{A.26}$$

Appelquist et al. (1974) estimate the electromagnetic to hadronic branching ratio directly by comparing two-photon emission with two-gluon emission:

$$\frac{\Gamma(\eta_c \to \gamma\gamma)}{\Gamma(\eta_c \to \text{hadrons})} = \mathcal{O}\left(\frac{\alpha^2}{\alpha_s^2}\right) \simeq 10^{-3} \quad . \tag{A.27}$$

With their estimate[5]

$$\Gamma(\eta_c \to \text{hadrons}) = 6.5 \text{ MeV}\ , \tag{A.28}$$

this implies

$$\Gamma(\eta_c \to \gamma\gamma) = \text{few keV}. \tag{A.29}$$

Vector dominance together with the coupling estimate (A.18) implies $\Gamma \sim \widetilde{\Gamma}/m$, which would seriously contradict the large $(\eta \to \gamma\gamma)/(\pi^0 \to \gamma\gamma)$ ratio. (See Browman, et al., 1974).

The estimate (A.26) may be large enough to permit the observation of the η_c using the Primakoff effect. Using the value $\Gamma(\eta_c \to \gamma\gamma) = 100$ keV, Lee and Quigg (1974) have predicted that on Pb,

$\sigma(\gamma \to \eta_c) \simeq 170$ nb at $E_\gamma = 100$ GeV. Hence, very roughly,

$$\sigma_{Pb}(\gamma \to \eta_c)/\mathrm{nb} \simeq 1.7\ \Gamma(\eta_c \to \gamma\gamma)/\mathrm{keV}, \tag{A.30}$$

since the cross section scales linearly with the $\gamma\gamma$ width. The coherent production cross sections on various targets are shown in Figs. A.1 as functions of the incident γ energy. The best channels for observing the η_c, as we have mentioned, would be $p\bar{p}$ and $\Lambda\bar{\Lambda}$. In analogy with the decay of the E(1420), we might also expect to see η_c in the $K^{\pm}\pi^{\mp}K_S^0$ channel. The mode $\pi^{\pm}A_2^{\mp}$ may also be important (French, 1968).

The excited states of the $c\bar{c}$ system have been discussed by many authors, including Appelquist et al., (1975); Callan et al., (1975); Eichten et al., (1975), and Schnitzer (1974). A rough guess as to their masses and decays is shown in Table A.2. (See Table A.4 for a wider range of possibilities.)

Table A.2

Excited states of the $c\bar{c}$ system

L = 0			L = 1			L = 2		
${}^{S}L_J$	J^{PC}	mass, MeV	${}^{S}L_J$	J^{PC}	mass, MeV	${}^{S}L_J$	J^{PC}	mass, MeV
${}^1S_0$	0^{-+}	3050	${}^3P_0$	0^{++}		${}^3D_1$	1^{--}	
${}^3S_1$	1^{--}	ϕ_c (3105)	${}^1P_1$	1^{+-}	~3400	${}^1D_2$	2^{-+}	~3700
${}^1S_0'$	0^{-+}	3650	${}^3P_1$	1^{++}		${}^3D_2$	2^{--}	
${}^3S_1'$	1^{--}	ϕ_c' (3695)	${}^3P_2$	2^{++}		${}^3D_3$	3^{--}	

All of the states in Table A. 2 are expected to lie below charm-anticharm threshold. Their electromagnetic decays thus can compete favorably with their hadronic decays: $\Gamma\left[(c\bar{c}) \rightarrow (c\bar{c})' + \gamma\right]$ should be of order several hundred keV for many of the transitions between the states in Table A. 2, while for the negative charge-parity states one expects $\Gamma(c\bar{c} \rightarrow \text{hadrons}) \leq \mathcal{O}\,\Gamma(\phi_c \rightarrow \text{hadrons}) = \mathcal{O}(100\ \text{keV})$ and for the positive charge-parity states $\Gamma(c\bar{c} \rightarrow \text{hadrons}) \leq \mathcal{O}\,\Gamma(\eta_c \rightarrow \text{hadrons}) =$ (few MeV). In specific dynamical models the suppression of the $L \neq 0$ $c\bar{c}$ wave functions will reduce the rates for decays into charmless hadrons even further.

The electromagnetic transitions among the states in Table A. 2 should result in numerous monochromatic photons with varied energies in the range of several tens to several hundreds of keV. The detection of such photons will be crucial both to verify the charm scheme and to determine the laws governing hadron structure. If the charm scheme is correct, the $c\bar{c}$ system is a unique gift of nature. Its study is likely to provide us with the long-sought (probably non-Coulombic) "Bohr theory" of the hadrons.

We have already discussed the decay $\phi_c \rightarrow \eta_c\gamma$. It is a quark-spin-flip (M1) transition. The decay $\phi_c'(3695) \rightarrow \eta_c\gamma$ also is an M1 transition. Estimates of its rate depend on the overlap between the n=1 and n = 2 wave functions, where n is the principal quantum number:

$$\Gamma(\phi_c' \to \eta_c \gamma) \quad \begin{array}{l} = 40 \text{ keV (Appelquist et al., 1975)}, \\ = 25 \text{ keV (Callan et al., 1975)}, \\ = 1 \text{ keV (Eichten et al., 1975)}. \end{array} \tag{A.31}$$

We recall that from experiment $\Gamma_{tot}(\phi_c') \leq 2.7$ MeV. If the first two estimates in (A.31) are closer to the truth, the best place to produce η_c will be in colliding $e^+ - e^-$ beams with $E_{CM} = 3.7$ GeV. If the smaller estimate is correct, one should turn to the Primakoff effect or some hadronic process (e.g., $\bar{p}p \to \eta_c$, $\pi^- p \to \eta_c n$, ...) to produce η_c.

Certain hadronic decays of the states in Table A.2 may proceed faster than others. One class of decays which may have already been observed in $(c\bar{c}) \to (c\bar{c}') + (2 \text{ gluons})_{\text{color singlet}}$. This could be the mechanism for the reaction (A.14), which accounts for a sizeable fraction of all the decays of the ϕ_c' (3695). According to Appelquist et al. (1975), a similar chain might be important for $\eta_c' \to \eta_c + 2\pi$.

Another class of hadronic decays which might not be negligible might be $(c\bar{c})_{C=+} \to (2 \text{ gluons}) \to$ hadrons. Appelquist et al. (1974) estimate this to be an important process only for the L = 0 states η_c and η_c'. Perhaps the two peaks seen by (Alexander et al., 1970) correspond to these states. However, if one admits appreciable effects from the gradient of the wave function at the origin, the L = 1 states of positive charge-parity can also decay in this manner. The position of the second peak seen by Alexander et al. (1970), at 3.4 GeV, is indeed consistent with that of the $(0^{++}, 1^{++}, 2^{++})$ group in Table A. 2.

Because of the likelihood that at least the 0^{-+} $c\bar{c}$ states (and possibly others) couple to hadrons with widths of the order of MeV, we urge the systematic study of $p\bar{p}$ spectra and $p\bar{p}$ direct channel processes in the interesting range $2.5\ \text{GeV} \leq E_{CM} \leq 4\ \text{GeV}$. A priori, the $p\bar{p}$ system can communicate with any $c\bar{c}$ system, and we have argued that it is most likely to do so for states of positive charge-parity (and hence positive G-parity).

The argument that charmless hadrons communicate with the charmonium ($c\bar{c}$) world via C = + states may help to explain why the MIT-Brookhaven group do not see the ϕ_c' in their experiment at anything greater than 1% of the rate of ϕ_c production (Leong, 1974). Suppose that the process they observe goes via the chain

$$
\begin{aligned}
p + \mathrm{Be} &\to (C = +, c\bar{c}\ \text{state}) + X \\
&\qquad \hookrightarrow \phi_c(3105) + (\gamma \text{ or hadrons}) \\
&\qquad\qquad \hookrightarrow e^+e^- \quad . \qquad\qquad \text{(A. 32)}
\end{aligned}
$$

The ratio of ϕ_c' to ϕ_c production will then depend on the spectrum of available C = + parent states and their branching ratios into the appropriate vector mesons. If the only available parent state for the ϕ_c' lies above charm-anticharm threshold, for example, the ϕ_c' will not be produced at all. Instead, the parent will decay strongly into a pair of charmed mesons. Apart from this, the small branching ratio of $\phi_c' \to e^+e^-$ is a major reason for the suppression.

IV. MASS ESTIMATES OF CHARMED PARTICLES

We now turn to a discussion of mass formulae. So far we have been occupied entirely with the $c\bar{c}$ system, and we now turn to the states ($c\bar{u}$, $c\bar{d}$, $c\bar{s}$ and their charge conjugates) and the baryonic states cuu, cud, and cdd. These states are the most likely to be observed in the near future, and the reader may deduce the consequences for others from the main body of our paper.

The mass formulae in our article are equivalent to the following simple quark-model rules for the charmed vector mesons $D^{*+} = c\bar{d}$, $D^{*0} = c\bar{u}$, $F^{*+} = c\bar{s}$ and their charge conjugates:

$$m^2_{D^*} = \frac{1}{2}\left(m^2_{\phi_c} + m^2_{\rho}\right) \Rightarrow m_{D^*} = 2.26 \text{ GeV},$$

$$m^2_{F^*} = \frac{1}{2}\left(m^2_{\phi_c} + m^2_{\phi}\right) \Rightarrow m_{F^*} = 2.31 \text{ GeV}. \qquad (A.33)$$

If one were to use instead a linear interpolation formula the masses of the charmed vector meson would be around 2 GeV.

To estimate the masses of the singly charmed pseudoscalar mesons one can use the analogue of (A.33), assuming some value for the mass of η_c. We shall take $m_{\eta_c} = 3.05$ GeV. We then obtain

$$m^2_D = \frac{1}{2}\left(m^2_{\eta_c} + m^2_{\pi}\right) \Longrightarrow m_D = 2.16 \text{ GeV},$$

$$m^2_F = m^2_D + m^2_K - m^2_{\pi} \Longrightarrow m_F = 2.21 \text{ GeV}. \qquad (A.34)$$

The mass of the D is compatible with the guess made above that charm-anticharm threshold lies below 4.7 GeV. If one uses linear interpolation one estimates $m_D = 1.6$ GeV. This lies below the bound set by the narrowness of the ϕ_c', $m_D \geq 1.85$ GeV.

A convenient mnemonic for $L = 0$ ground state meson masses roughly equivalent to the above estimates is

$$m^2_{(\mathrm{GeV}^2)} = 0.02 + 0.23 n_s + 4.53 n_c + 0.56 S_q \qquad (A.35)$$

where n_s is the number of strange quarks, n_c is the number of charmed quarks, and S_q is the quark spin (0 or 1). We must stress that all these estimates apply standard first-order symmetry breaking to much greater splittings than those encountered previously. Hence we should not be at all surprised if our charmed particle mass estimates were off by as much as one or two hundred MeV. There is no substitute for dynamical calculations, which we do not perform.

From the predicted mass of the D in (2.16) one can obtain the parameter R introduced in (3.2):

$$R \equiv \frac{m_c - m_u}{m_s - m_u} = \frac{m_D^2 - m_\pi^2}{m_K^2 - m_\pi^2} \simeq 20 \ . \qquad (A.36)$$

This value permits us to estimate the masses of the charmed baryons, using the linear formulae (3.9) and their analogue for $3/2^+$ states. We can also estimate baryon masses by assuming that (3.9) applies to squares of masses, and finally, we can try linear formulae for both mesons and baryons. The results are shown in Table A. 3.

Table A. 3

Attempts to guess the mass of charmed baryons

Meson mass formula	Baryon mass formula	R	$m_{C_1^{++}}$ (cuu, $\frac{1}{2}^+$)	$m_{C_0^+}$ (c[ud], $\frac{1}{2}^+$)	$m_{C_1^{*++}}$ (cuu, $3/2^+$)
quadratic	linear	20	6	4.4	4.2
quadratic	quadratic	20	3.4	$\underline{2.8}$	3.1
linear	linear	10	3.5	2.7	2.7

The underlined state in Table A. 3 is stable with respect to strong and electromagnetic decays. Its favored two-body nonleptonic modes (no $\sin\theta_c$ factors) are $\Lambda\pi^+$, $\Sigma^0\pi^+$, $\Sigma^+\pi^0$ and $p\bar{K}^0$. It has two-body nonleptonic modes with a $\sin^2\theta_c$ suppression consisting of $n\pi^+$, ΛK^+, $\Sigma^0 K^+$ and $\Sigma^+ K^0$. Given our estimate (A. 34), <u>any</u> charmed baryon below 3 GeV should be stable with respect to strong and electromagnetic decays. (The states on the last line of Table A. 3 will decay strongly if Eqs. (A. 34) are replaced by linear formulae.)

One oddity of Table A. 3 is the inversion of the C_1^{++} $(1/2^+)$ and $C_1^{*++}(3/2^+)$ masses with respect to (say) the $\Sigma(1/2^+)$ and $Y_1^*(3/2^+)$ masses. If this could be confirmed, it would be dramatic evidence for first-order symmetry breaking, to say the least. More likely, none of the entries in Table A. 3 is particularly correct, and one might just as well estimate charmed baryon masses by adding about $1\frac{1}{2}$ GeV ($m_{\phi_c}/2$) to the correspondin ones for charmless baryons:

$$m_{C_1^{++}} \simeq m_{\Sigma^+} + 1.5\ \text{GeV} \simeq 2.7\ \text{GeV}$$

$$m_{C_0^+} \simeq m_{\Lambda} + 1.5\ \text{GeV} \simeq 2.6\ \text{GeV}$$

$$m_{C_1^{*++}} \simeq m_{Y_1^*} + 1.5\ \text{GeV} \simeq 2.9\ \text{GeV}. \qquad \text{(A.37)}$$

In this case the doubly-charged, singly-charmed baryons also become (meta) stable. Their weak nonleptonic decays include $\Sigma^+\pi^+$, $\Lambda\pi^+\pi^+$ and $p\bar{K}^0\pi^+$ (not suppressed by $\sin^2\theta_c$ factors, but possibly suppressed since these are exotic channels[8]), and $p\pi^+$ and Σ^+K^+ (suppressed by $\sin^2\theta_c$).

A comment on nonleptonic decays of charmed particles: According to the enhancement mechanism alluded to in the text (Sec. 4.2), the enhanced piece of the $\Delta C = \Delta S = 1$ interaction has the form $(\bar{s}c)(\bar{u}d) - (\bar{u}c)(\bar{s}d)$, and therefore has $\Delta V = 0$. (Recall V-spin acts on u and s). The D^+ has $V = 0$, so its decay into $\bar{K}^0\pi^+$, whose symmetric S-wave combination has $V = 1$, is forbidden in the SU(3) limit (see also Kingsley, et al., 1975; for more general and exhaustive considerations based on SU(3)). However $D^\pm \to K\pi\pi$ is in general allowed; we do not think that the total decay rate of $D^\pm$ is suppressed relative to that of $D^0, \bar{D}^0$ by more than a factor of 2, say. After all, though, the enhancement of nonleptonic weak interactions might arise from a quite different source (see, for example, Lee and Treiman, 1971); it is possible that nonleptonic decays of charmed particles arise effectively from the metamorphosis $c \to u$. (We thank J. D. Bjorken for reminding us of this). In such a case, the D-mesons will decay mostly into nonstrange hadrons, and the F-mesons into strange ones.

If the charmed baryons are all unstable with respect to decay into ordinary baryons and charmed mesons, their identification may be very difficult. Nonetheless, the discovery of the resonances at 3100 and 3700 MeV, and their identification with $c\bar{c}$ states, has considerably reduced the highest mass at which we expect the lowest charmed baryon to occur: from 19 GeV (see Sec. 3) to around 4 GeV.

The orbital and radial excitations of charmed mesons are of interest primarily as an aid to Regge phenomenology. If the intercepts of the trajectories of the D^* and its tensor partner lie high enough, associated production reactions such as

$$\pi^- + p \to M_c + B_c \tag{A.38}$$

may not be suppressed at high energy as strongly as indicated in Sec. 5.5.

An optimistic estimate of the D^* intercept has been obtained by Field and Quigg (1975). Let us denote the 2^+ partner of the D^* as D_T^*. If one is entitled to use Eq. (3.8) for tensor mesons with the <u>same value of R</u>, one finds

$$\frac{m_{D_T^*}^2 - m_{A_2}^2}{m_{K^{**}}^2 - m_{A_2}^2} = \frac{m_{D^*}^2 - m_\rho^2}{m_{K^*}^2 - m_\rho^2} = \frac{\frac{1}{2}(m_{\phi_c}^2 - m_\rho^2)}{\frac{1}{2}(m_\phi^2 - m_\rho^2)}$$

i.e.,

$$m_{D_T^*}^2 = 8.29\ \mathrm{GeV}^2 \quad \text{or} \quad m_{D_T^*} = 2.88\ \mathrm{GeV}\ . \tag{A.39}$$

Further assuming the D^* and D_T^* to lie on a single exchange-degenerate trajectory, one finds this trajectory to be

$$\alpha_{D^*}(t) = -0.61 + 0.32\,t\ . \tag{A.40}$$

This exercise clearly depends on taking seriously the small discrepancy between $m_{K^*}^2 - m_\rho^2 \simeq 0.21$ and $m_{K^{**}}^2 - m_{A_2}^2 \simeq 0.30$; its validity is probably no greater than the baryon predictions of Table A.3. Given $m_{D_T^*}$, we can then estimate the mass of $\phi_{cT}(2^+)$ using

$$m_{\phi_{cT}}^2 = 2m_{D_T^*}^2 - m_{A_2}^2 \tag{A.41}$$

and find

$$m_{\phi_{cT}}^2 = 15.17\ \mathrm{GeV}^2;\ \ m_{\phi_{cT}} = 3.87\ \mathrm{GeV} \tag{A.42}$$

corresponding to a trajectory (assumed exchange-degenerate)

$$\alpha_{\phi_c}(t) = -0.79 + 0.19\,t\ . \tag{A.43}$$

The F^* trajectories should be fairly close to the D^* trajectories in any model, and we shall not estimate them separately.

The next estimate we can give for Regge trajectories (always assuming a straight-line form, which may be questionable) is based on taking the harmonic-oscillator spectrum, for which ϕ_c' is degenerate with L = 2 excitations. In this case the ϕ_c trajectory has roughly half the usual slope. One interpolates for the D_T^* mass using (A.41) to find the D^* trajectory. Finally, one can assume the usual slope $\alpha' \simeq 0.9\ \mathrm{GeV}^{-2}$. (Dual models for $\pi\pi \to D\bar{D}$ require charmed particle trajectories to have the same slope as charmless ones, as pointed out to us by P. Freund.) The results of all three methods are collected in Table A.4.

Table A.4

Various estimates of charmed particle Regge trajectories.
(For the description of different methods used, refer to the text.)

Method	α_D^*	$m(D_T^*) \simeq m(F_T^*)$ [a] in GeV.	$\alpha_{\phi_c}(t)$	$m(\phi_{c\,T})$ [a] in GeV.
1	$-0.6 + 0.32\,t$	2.9	$-0.79 + 0.19t$	3.9
2	$-2.4 + 0.66\,t$	2.6	$-3.8 + 0.50\,t$	3.4
3	$-3.6 + 0.9\,t$	2.5	$-7.7 + 0.9\,t$	3.3

[a] The mass of tensor meson (2^{++}). There may also be 0^{++}, 1^{++}, 1^{+-} mesons nearly degenerate with 2^{++}.

One might expect the estimates of the 2^+ masses given here to be valid for all of the L = 1 states: ${}^3P_0$, ${}^3P_1$, and ${}^1P_1$ as well as ${}^3P_2$. For example, one might expect to be able to produce an axial-vector F_A^{*+} (the analogue of the elusive A_1) in the diffractive reaction

$$\underset{(\text{target})}{\nu + p} \to \mu^- + F_A^{*+} + \underset{(\text{slow})}{p} \quad . \tag{A.44}$$

A likely mass for this F_A^* on the basis of Table A.4 would be 2.6 to 2.7 GeV. Its most likely decay would be to $F^+(2.21) + \gamma$, and the F^+ could be detected by its nonleptonic mode as a peak in effective mass (e.g., of three charged pions).

The L-excited D^* states in Table A.4 contain nonstrange quarks. They are allowed to decay to $D^*\pi$ and/or $D\pi$ both by Zweig's rule and by phase space, and presumably do so most of the time. By contrast, the rates for $F^*(L = 1) \rightarrow \{F^*(L = 0) + 2\pi \text{ or } F(L = 0) + 2\pi\}$ are expected to be at least as small as that for $\phi_c' \rightarrow \phi_c + 2\pi$, and other hadronic channels are probably closed. If one can ever produce them, the lowest excited states of the F should bear some resemblance to charmonia ($c\bar{c}$ systems).

V. ESTIMATES OF PRODUCTION CROSS SECTIONS

The mass scale set by the ϕ_c and ϕ_c' allows us to make firmer estimates of production processes. We turn first to the photoproduction of these two vector mesons.

In Sec. 5.5 we estimated a differential cross section for ϕ_c photoproduction at $t = 0$ of $\sim 40\ \mu b/\text{GeV}^2$. With a slope at 200 GeV of about 6 GeV^{-2} (see Moffeit, 1973), this would imply a total cross section of 6-7 μb for ϕ_c photoproduction. This is many times larger than the present experimental upper limit (M. Perl, private communication):

$$\sigma(\gamma N \rightarrow \phi_c N) \lesssim 30 \text{ nb} \quad , \tag{A.45}$$

but several factors could work to reduce our estime. (1) If one simply assumes $\sigma_t(\phi_c N)_{200\ \mathrm{GeV}/c} \simeq \sigma_T(\phi N)_{10\ \mathrm{GeV}/c}$ and uses data on $\gamma N \to \phi N$ and the coupling estimate (A.15), one obtains[10]

$$\left.\frac{d\sigma}{dt}\right|_{t=0} (\gamma N \to \phi_c N) = \frac{\gamma_\phi^2}{\gamma_{\phi_c}^2} \left.\frac{d\sigma}{dt}\right|_{t=0} (\gamma N \to \phi N): \tag{A.46}$$

$$= 4(2.85 \pm 0.2\ \mu b/\mathrm{GeV}^2) \simeq 11\ \mu b/\mathrm{GeV}^2 .$$

The larger value in Sec. 5.5 depends on using (5.5) for the total ϕN cross section, which is larger than the value implied by present low-energy ϕ photoproduction data. (2) The square of the photon -ϕ_c coupling, evaluated at $m_{\phi_c}^2 = q^2$, seems to be about a factor of 3 less than that estimated from (A.15). If this same suppression holds at $q^2 = 0$, one might expect a corresponding suppression factor (about 3) in the cross section for ϕ_c photoproduction. (3) A large class of models for the Pomeranchuk trajectory (see, e.g., Carlitz, et al., 1971) predicts the suppression of Pomeron couplings to particles containing strange or charmed quarks. For example, such models predict the asymptotic ratio of $\phi_c N$ to ϕN total cross sections to be

$$\left[\frac{\sigma_T^{\mathrm{Pom.}}(\phi_c N)}{\sigma_T^{\mathrm{Pom.}}(\phi N)}\right]^2 = \left[\frac{\alpha_{\mathrm{Pom.}}(0) - \alpha_{f'}(0)}{\alpha_{\mathrm{Pom.}}(0) - \alpha_{\phi_c T}(0)}\right]^2$$

$$\simeq \frac{1}{10} \text{ to } \frac{1}{75} \tag{A.47}$$

where the estimates of the ϕ_{cT} trajectory intercept are taken from Table A.4. These ratios may also have to be applied to our prediction. To summarize, a likely range for the total ϕ_c production cross section is

$$\begin{aligned}\sigma(\gamma N \to \phi_c N) &= (\text{Initial estimate}) \\ &\times (\text{Photon coupling suppression}) \\ &\times (\text{Pomeron coupling suppression}) \\ &= (2 \text{ to } 7\ \mu\text{b}) \times (\tfrac{1}{3}) \times (\tfrac{1}{10} \text{ to } \tfrac{1}{75}) \\ &= 10 \text{ to } 200 \text{ nb}\end{aligned} \tag{A.48}$$

if the Pomeron models of Carlitz et al. (1971)("f-dominated Pomeron") are correct. This range is consistent with the estimate (A.45), and probably also with the forthcoming result from Fermilab (W. Lee et al., private communication)

Using Eq. (A.20), we would estimate the cross section for ϕ_c' photoproduction to be about a factor of 2 less than that for ϕ_c.

If one assumes that the photon-ϕ_c coupling does not vary too much between $q^2 = m_{\phi_c}^2$ and $q^2 = 0$, the photoproduction of ϕ_c becomes an experiment to measure the Pomeranchukon coupling to ϕ_c. This is of tremendous importance in estimating the production of charmed particle pairs. In the central region of rapidity space, the asymptotic rate of production of any particle A is governed by the forward three-particle-to-three particle amplitude with two Pomeranchukon (Fig. A.1): A particular model for the Pomeranchuk trajectory considered by Farrar and Rosner (1974), generalizing an approach by Cahn and Einhorn (1971), relates the coupling of particles in Fig. A.1 to Pomeron-Pomeron-particle couplings in such a way that

$$\left.\frac{\sigma(D)}{\sigma(K)}\right|_{\text{central}} = \frac{\sigma_T^{\text{Pom}}(\phi_c N)}{\sigma_T^{\text{Pom}}(\phi N)}$$

$$\sim \frac{1}{3} \text{ to } \frac{1}{9} \qquad \text{(A.49)}$$

if we use (A.47). These are enormous values for charmed particle production. They conflict strongly with thermodynamic estimates based on the formula of Hagedorn (1971):

$$\sigma(M_\perp) \sim e^{-M_\perp / T_0}$$

$$M_\perp \equiv (p_\perp^2 + M^2)^{1/2}$$

$$T_0 \sim 160 \text{ MeV} \ . \qquad \text{(A.50)}$$

However, the asymptotic limits (A.49) are not likely to be approached until well above ISR energies in a multiperipheral model (e.g., Einhorn and Nussinov, 1974). This is because charmed particles are presumably produced in pairs, with a cluster mass of at least 2 m_D = 4 to 5 GeV, and the production of such a massive cluster is highly disfavored in multiperipheral models because of t_{min} effects. While such effects are hard to estimate, they could easily increase the cross section for charmed particle production in the central region at Fermilab and CERN II by

10 - 100 (or even greater) over that at Brookhaven and the CERN PS.

Estimates for charmed particle production via the two-gluon production of a $c\bar{c}$ pair have been made by Einhorn and Ellis (1975). These calculations are very sensitive to the assumed gluon spectrum in a hadron. If one takes the gluons to have the same x-distribution in the hadron as the anti-partons, for example (i.e., peaked toward low x), the cross section for $c\bar{c}$ production can rise by several orders of magnitude between Brookhaven and ISR energies.

Let us now estimate the cross section for charmed particle production at Fermilab energies (150 ~ 300 GeV for protons) in the diffractive region, as mentioned at the end of Sec. 5. The minimum mass of a diffractively produced cluster in the reaction

$$\begin{array}{l} \text{Meson} + \text{Target} \to (M^*) + \ldots \\ \text{(or photon)} \qquad\qquad \hookrightarrow M_c\ \bar{M}_c \end{array} \tag{A.51}$$

is at least $2\,m_D \simeq 4$ GeV, and for a baryonic cluster in

$$\begin{array}{l} \text{Baryon} + \text{target} \to (N^*) + \ldots \\ \qquad\qquad\qquad \hookrightarrow B_c\ M_c \\ \qquad\qquad\qquad \hookrightarrow B\ M_c\ \bar{M}_c \end{array} \tag{A.52}$$

is probably $m_p + 2m_D \simeq 5$ GeV. Let us assume that the diffractive process is important only for $x = 1 - M^2/s \gtrsim 0.9$, where M is the mass of the cluster. (See, e.g., the review by Leith, 1973). Then for the process (A. 52) the interesting range is $25 \leq M^2 \leq 60$ GeV^2, for which we estimate the diffractive cross section at a single vertex to be of the order of several hundred microbarns. Of course, either the target or the projectile can undergo diffraction in these processes. Projectile diffraction is ideal for observing short tracks, while target diffraction allows better resolution in plotting effective masses. Observing the target recoil also provides a handle for measuring the effective mass of the cluster.

Given an N^* cluster with masses squared around 40 GeV^2, what is its probability of undergoing the decays mentioned in (A. 52)? Such a cluster can also be produced in the direct channel by pions of about 20-25 GeV/c on nucleons (and this might be as good a source of charmed particles as high-energy diffraction). As a rule of thumb, noting that $m_c : m_s \simeq m_s : m_u$, we shall guess that

$$\frac{\sigma(\text{charmed})}{\sigma(\text{strange})} \simeq \frac{\sigma(\text{strange})}{\sigma(\text{total})} \simeq 10\text{-}15\% \quad \begin{array}{l}\text{in } \pi^{\pm}p \text{ interactions} \\ \text{at 20-25 GeV/c.}\end{array} \tag{A. 53}$$

Combining this estimate with the estimate of several hundred μb for diffractive N^* production in the mass range of interest, we arrive at an estimate of several μb for charmed particle production in the reaction (A. 52). This would correspond to one charmed particle event in several thousand in the emulsion exposures at Fermilab as mentioned at the end of Sec. 6. If marked by a distinctive signature, such as a forward-going doubly-charged track (as one would have if the C_1^{++} were stable), such events would not be too hard to identify. Note that we have argued that the C_1^{++} might live longer than the average charmed particle: even for $m = 3$ GeV, a lifetime of 10^{-13} sec would not seem unreasonable.

If the estimate in (A. 53) is really correct, of course, pion-nucleon interactions at 20-25 GeV/c become an ideal place to look for charmed particles. We would imagine that Eq. (A. 53) should really be applied, in pion-nucleon interactions, to the non-Pomeron contribution to the total cross section, since associated production will be the main mechanism for production of charmed particles at such low energies. This non-Pomeron contribution is of the order of a couple of milli-barns, which still allows for charmed particle production cross sections of the order of several tens of microbarns. Typical reactions to look for would be

$$\pi^- + p \to \underset{\substack{[d\bar{c}] \\ \hookrightarrow K^0\pi^-}}{D^-} + \begin{Bmatrix} C_0^+ \\ \underset{\hookrightarrow \Lambda\pi^+}{C_1^+} \end{Bmatrix} \tag{A. 54}$$

or

$$\pi^+ + p \to \underset{\substack{[u\bar{c}] \\ \hookrightarrow K^+\pi^-}}{\bar{D}^0} + \underset{\hookrightarrow \Lambda\pi^+\pi^+}{C_1^{++}} \quad . \tag{A. 55}$$

We remind the reader that the best channels in which to make effective-mass studies may be non-exotic ones (those with the quantum numbers of

K^0, such as $K^+\pi^-$ and $K_s\pi^+\pi^-$, or Σ^+, such as $\Lambda\pi^+$ and $p\bar{K}^0$). The baryons in Eqs. (A. 54) and (A. 55) may also be unstable with respect to the strong interactions, decaying to charm = +1 mesons (which give rise to S = -1, preferentially) and ordinary baryons.

If one were to estimate particle production cross sections using the formula (Snow, 1973) $\sigma(x) \sim 1/M_x^2$, the above estimates for diffractive and associated production would read, respectively,

$$\sigma_{diff}(\text{charm}) \sim \frac{m_\pi^2}{m_D^2} \text{ (few hundred } \mu\text{b)}$$

$$\sim \mathcal{O}(1\,\mu\text{b})$$

$$\text{around } p_L = 300 \text{ GeV/c} \tag{A. 56}$$

and

$$\sigma_{assoc}(\text{charm}) \sim \frac{m_\pi^2}{m_D^2} \text{ (couple of mb)}$$

$$\sim \text{several } \mu\text{b}$$

$$(\pi p \text{ interactions around } p_L = 20 \text{ GeV/c}). \tag{A. 57}$$

Even with such optimistic estimates, the identification of charmed particles from mass spectra in bubble chamber experiments would be marginal. If the exponential spectrum in (A. 50) is closer to the truth, one will have to rely exclusively on high-statistics, high-resolution counter experiments for such studies. The estimates of Field and Quigg (1975) on two-body associated charm production cross sections based on

Regge pole phenomenology are far smaller than (A. 57) even with the most optimistic Regge trajectory.

What about $e^+ - e^-$ reactions? If the excess in R above our lower curve in Fig. 8 is really due to charm production, we would expect to see charmed particles with a cross section equal to this excess.

One possibility, which we feel deserves some study, is that between the mass of the ϕ_c' and charm threshold the excess above the three-quark value of R is due to <u>nonresonant</u> $c\bar{c}$ production. In that case one would expect the $c\bar{c}$ system to radiate gluons or photons until it reached a narrow resonant state :

$$\begin{aligned} e^+ + e^- \to c\bar{c} &\to (\phi_c \text{ or } \phi_c') + \gamma' s \\ \text{or } &\to (\phi_c \text{ or } \phi_c') + (\text{gluons}) \\ &\qquad\qquad\qquad\quad \hookrightarrow \text{pions} \;. \end{aligned} \tag{A. 58}$$

In this case the <u>inclusive</u> production of ϕ_c or ϕ_c' might be very large through a wide range of colliding beam energies. It might even predominate over the pair production of charmed particles until somewhat above charm threshold. At $E_{cm} = 4.8$ GeV, though, there seems to be no sign of a ϕ_c recoiling against $\pi^+\pi^-$ (J. D. Jackson, private communication).

The more straightforward charm-related explanation for the excess of R above the three-quark value would, of course, be the pair production of charmed particles: just above threshold,

$$e^+ + e^- \to D^0 + \bar{D}^0 \tag{A.59}$$

$$\to D^+ + D^- \tag{A.60}$$

$$\to F^+ + F^- \tag{A.61}$$

with single- or double-vector meson production and inclusive channels becoming important by a few hundred MeV above threshold. In the exact U(4) limit, as we have mentioned, reaction (A.59) should be suppressed; the contributions of the respective quark charges cancel. This mechanism would also suppress D^{0*} - $\bar{D}^{0*}$ production, though not D^0 - $\bar{D}^{0*}$ production (or its charge conjugate). The rates for (A.60) and (A.61) would be equal. Now, we have argued that since the non-$\sin^2\theta_c$ two-body decay of the charged D may be suppressed, perhaps even to the level of the $\sin^2\theta_c$ decay (which involves channels with the quantum numbers of the singly-charged pion). The favored decay of the F also involves quantum numbers of the singly-charged pion. Consequently, in the U(4) limit, the best place to look for charmed particles in colliding beam experiments may be in such mass combinations as $(3\pi)^\pm$, $K^\pm K_S$, $\eta\pi^\pm$ and so on. High resolution will be essential to avoid background problems. If the enhancement mechanism of nonleptonic interactions is not what we envisaged here, and if it results in the effective $c \to u$ conversion, then the strange particle yield will not increase above the $D\bar{D}$ threshold. It will, somewhat, only after the $F\bar{F}$ threshold is reached.

If one notes that U(4) is badly broken since the ϕ_c pole lies much closer to the physical region for the reactions (A.59 - A.61) than do the other vector meson poles, the reaction (A. 59) is not suppressed as much. It is still expected to be less frequent than (A. 60) or (A. 61), however. For production of a pseudoscalar-vector pair, the roles are reversed; the analogue of Eq. (A. 59) would dominate the analogues of Eq. (A. 60) and (A. 61), strongly in the exact U(4) limit (by a factor of 16!) and considerably less so if the ϕ_c pole dominated charmed particle production.

At the very least, we would regard the absence of a charmed particle signal in the $K^{\pm}\pi^{\mp}$ channel in colliding-beam data around 5 GeV as evidence that our Table IV of branching ratios is unreliable. If a $K^{\pm}\pi^{\mp}$ signal is not even seen at the level of a few percent of all kaon-containing hadronic events at $E_{cm} \sim 6$ GeV, we would begin to suspect the validity of the charm hypothesis $\phi_c = \psi$, J itself.

The photoreaction

$$\gamma + N \rightarrow (\text{Charm}) + \overline{(\text{Charm})} + N \tag{A.62}$$

bears the same relation to colliding $e^+ - e^-$ beam experiments as the diffractive processes (A. 51) and (A. 52) bear to associated production (e.g., A. 38). One would expect the cross section for reaction (A. 62) above charm threshold to be of the same order as $c\bar{c}$ vector meson production, as this reaction is likely to be dominated by the nearby ϕ_c and ϕ_c' poles. Hence, using (A. 48) and the fact that total photon-nucleon cross sections are of the order of 100 μb, one might expect 10^{-4} to 2×10^{-3} of the photoreactions at high energies to involve the process (A. 62). While this is not a particularly large number, the reaction (A.62) may have some intrinsic advantages, for example in emulsions where the use of a neutral beam avoids large numbers of non-interacting tracks. (As we have stressed, the major use of emulsions is in detecting short tracks.)

We would like to add some remarks concerning our estimates of charmed particle production in neutrino reactions (Sec. 5.2). These remarks are based on the mass scale for charmed particles implied by taking the resonances at 3100 and 3700 MeV to be the ϕ_c and ϕ_c'.

The process illustrated in Fig. 6 and 7 was expressed in parton language as occurring via the transformation of a strange quark or antiquark (s or $\bar{s}$) in the $q\bar{q}$ "sea" of the target nucleon into a charmed quark or antiquark (c or $\bar{c}$). From Fig. 6 one can see that charm production should account for roughly 10% of deep inelastic antineutrino-nucleon interactions under the conditions shown. Since one expects the total deep inelastic charm production cross sections to be equal for neutrinos and antineutrinos, a few percent of neutrino deep inelastic interactions should contain charmed particles.

The above estimates may be viewed as reflecting cross sections for "inclusive diffractive production" of the states $(c\bar{s})^+$ (by neutrinos) or $(\bar{c}s)^-$ (by antineutrinos) off a nucleon target. Indeed, the dynamical assumptions that go into Fig. 6 (strong peaking toward $x = 0$, peaking toward $y = 1$) are just those that arise from t_{min} effects, which one would expect to be important in diffractive processes. If the production of charmed particles by neutrinos or antineutrinos is really diffractive, of course, the target should have a small recoil momentum, less than a GeV/c.

The expression for t_{min} in a deep inelastic process (see Sec. 5.2 for kinematic definitions) may be written

$$-t_{min} \simeq \left[\frac{M^2 - q^2}{2\nu}\right]^2 = \left[m_p x + \frac{M^2}{2y\, E_\nu}\right]^2, \tag{A.63}$$

where M is the mass of the diffractively produced state coupling to the current. If the cross section for such a process is peaked in t, where t is the momentum transfer between the current and the state of mass M, Eq. (A.63) can introduce considerable peaking toward y = 1.

For low-M^2 states, only the first term in (A.63) is important. This term is probably already taken into account in the phenomenological parton distributions that describe low-x behavior. For high-M^2 states, the cross term and the square of the second term in (A.63) become important. A calculation was thus performed in which the shape of the x-y distribution was described by

$$\frac{d\sigma^{(\nu,\bar{\nu})}}{dxdy} = \text{const.} \times 0.2\,(1-x)^7 \exp(b\,\tilde{t}_{min}) \tag{A.64}$$

$$b = 10\ \text{GeV}^{-2} \tag{A.65}$$

$$-\tilde{t}_{min} = (m_p x)\left(\frac{M^2}{yE_\nu}\right) + \left(\frac{M^2}{2y\,E_\nu}\right)^2. \tag{A.66}$$

where the only modification with respect to the original estimate in Sec. 5 is the exp $(b\,\tilde{t}_{min})$ factor.

The qualitative effects of the $\tilde{t}_{min}$ factor are roughly equivalent to choosing the rather high threshold of $m_c \simeq 5$ GeV as done in Sec. 5.2. Note that the absolute threshold for the diffractive process we are considering is only $m_F + m_p \simeq 3.2$ GeV. For $E_\nu \simeq 25$ GeV, most of the events in (A.64) occur with muon energies between 2 and 10 GeV, and muon angles with $\cos\theta_\mu \geq 0.98$. This suggests that the diffractive charm production process can probably be enhanced by cutting the data in $v = xy = E_\mu(1 - \cos\theta_\mu)/m_p$ and selecting(say) $v \lesssim 0.1$. For charm production in heavy nuclei, where the diffractive slopes are expected to be greater than (A.66), we expect the t_{min} effects to be correspondingly greater, and events will be peaked more strongly toward low x and high y. It is even conceivable that one could sort out such effects by looking at differences between neutrino-induced events in materials of two different atomic numbers.

If it is correct that charm production by neutrinos and anti-neutrinos can be viewed primarily as a diffractive effect, there are other diffractive effects that should be at least as great, such as three-charged pion production in the mass region 1100-1400 MeV (the A_1 region):

$$\nu(\bar{\nu}) + N \to \mu^-(\mu^+) + (3\pi)^+\left[(3\pi)^-\right] + N \ . \qquad (A.67)$$

There are several reasons why (A.67) should have a larger cross section than the corresponding charm production process

$$\begin{array}{l} \nu(\bar{\nu}) + N \to \mu^-(\mu^+) + (c\bar{s})^+ \left[(s\bar{c})^-\right] + N \\ \qquad (s\bar{c})^- \to \cdots \\ \qquad (c\bar{s})^+ \to F^+ + \gamma \\ \qquad (c\bar{s})^+ \to F^{*+} + \gamma \\ \qquad (c\bar{s})^+ \to D^0\, K^+ \\ \qquad (c\bar{s})^+ \to \cdots \end{array} \quad . \qquad \text{(A. 68)}$$

(1) The lowest mass state in (A. 68) may be as much as twice as heavy as that in (A. 67): We have argued earlier in this addendum, for example below (A. 44), that an axial vector F_A^* would have a mass of 2.6 to 2.7 GeV. The t_{min} effects discussed above will thus be much more important for (A. 68) than for (A. 67). (2) The effective coupling of the weak current to the $c\bar{s}$ system may be smaller than that to the $u\bar{d}$ system. (3) The Pomeranchuk trajectory may couple less strongly to the $c\bar{s}$ system than to the $u\bar{d}$ system. For $E_\nu = 25$ GeV, a rough estimate of these effects gives

$$\frac{\sigma(F_A^*)}{\sigma(A_1)} = \left(\frac{1}{2}\right)_{t_{min}} \left(\frac{1}{3}\right)_{\substack{\text{current}\\ \text{coupling}}} \left(\frac{1}{4}\right)_{\substack{\text{Pomeron}\\ \text{coupling}}} \simeq 4\%. \qquad \text{(A. 69)}$$

As one expects "A_1" production to be less than the total production of charmless hadrons in deep inelastic processes, the estimate (A. 69) may be somewhat more pessimistic than those based on the parton picture (sec. 5.2, Figs. 6 and 7).

An estimate (CERN Boson Workshop, 1974) of ϕ_c diffractive production in ν neutral currents gives a very small cross section.

The fact that charmed particles are expected to be produced in neutrino and antineutrino reactions with frequency of order several percent means that neutrino experiments are the best ones for emulsion exposures. As we have pointed out, it is only by the detection of short tracks in emulsions that one will be able to tell that a state is present which must decay weakly. Given our mass estimates and Fig. 10, the tracks of charmed particles at present-day energies will be too short to see in bubble chambers, but should definitely be of the order of tens or hundreds of microns: easily detectable in emulsions.

VI. SUMMARY

Let us summarize this addendum. The identification of the states at 3100 and 3700 MeV with ϕ_c and its first radial excitation narrows considerably the search for charm. If this identification is correct, one is in a much better position than six months ago to propose experiments which will confirm or rule out the charm idea. Detection of short tracks remain the crucial experiment, and becomes feasible now that the hypothetical charmed particle mass scale has been set. In addition, one must prove that the charmed quark couples more strongly to the strange quarks than to the quark d by a factor of cot θ. (Gell-Mann, 1964; Bjorken and Glashow, 1964). This will require the observation, for example, both of the decay

$$\bar{D}^0 \to K^+\pi^- \quad (\sim \cos^4 \theta) \tag{A.70}$$

and of

$$\bar{D}^0 \to \pi^+\pi^- \quad (\sim \sin^2 \theta \cos^2 \theta) \tag{A.71}$$

or of the pair

$$\bar{D}^0 \to K^+ \ell^- \nu \quad (\sim \cos^2 \theta) \tag{A.72}$$

and

$$\bar{D}^0 \to \pi^+ \ell^- \nu \quad (\sim \sin^2 \theta) \quad . \tag{A.73}$$

As nonleptonic enhancement effects are still not totally understood, the processes (A. 72) and (A. 73) may be more reliable for such a test. The process (A. 72) can lead to charged kaon-lepton coincidences, themselves a powerful indication in favor of charm. The expected kaon-lepton effective mass spectrum in the decay (A. 72) is shown in Fig. A. 3; it appears even possible to determine the mass of the parent through the detection of this spectrum.

The new resonances may, after all, turn out not to be associated with charm. However, in pursuing the experiments we have suggested, we suspect that more new effects are bound to show up. The emerging pattern of the hadrons is likely to be at least as interesting and varied as that we have described here.

The subject of this addendum has been the source of lively discussions with our theoretical and experimental colleagues. We would particularly like to thank F. di Bianca, D. Cline, A. Erwin, G. Feldman,

G. Goldhaber, J. Lord and F. Vanucci for their patience in explaining to us current limits on the effects we have mentioned. We would like to thank especially J. D. Jackson for sharing his knowledge and wisdom with us, and having gone through our first draft. Sam Treiman's encouragement has a lot to do with our undertaking this somewhat quixotic attempt at an instant review. We thank the members of the Theoretical Physics Department at Fermilab for discussions and enlightenment.

FOOTNOTES

[1]We urge the reader to consult the literature for the latest values for such quantities, as they are likely to be revised somewhat as more data accumulate.

[2]This rule has come to be known as "Zweig's rule" on the basis of partly oral tradition. It was first stated explicitly by (Okubo, 1963). It follows naturally in many dual models, for example in any theory which describes the decay of a resonance by the fissioning of a string into two strings.

[3]In this picture, the Coulombic interaction is mediated by "weakened" color gluon exchange. In the nonrelativistic approximation, the $c\bar{c}$ annihilation takes place at the origin, so the probability of annihilation is proportional to the square of the wave function at the origin which is proportional to the sixth power of the coupling constant. This depletion of the wave function at the origin is a direct consequence of the rather large spatial size of a Coulombic system. For phenomenological purposes, it suffices in most cases to assume that the $c\bar{c}$ wave function is small at the origin, for some reason.

[4]As the present article is meant primarily as a guide to experimentalists, we must reluctantly omit a large number of references to theoretical papers which perform this and similar calculations based on the direct

extension of well-known principles.

[5]One such model, considered by (Eichten, et al., 1975) uses (A.17) as an input which determined the square of the $c\bar{c}$ wave function at the origin.

[6]In positronium, $\Gamma(^1S_0)/\Gamma(^3S_1) = 9\pi/4(\pi^2 - 9)\alpha \simeq 1115$, a substantial enhancement over the expected scale of $\alpha^{-1} = 137$. A related enhancement is expected for charmonium (see Appelquist and Politzer, 1975): $\Gamma(^1S_0)/\Gamma(^3S_1) = 27\pi/5(\pi^2 - 9)\alpha_S \simeq 65$ for $\alpha_S = 0.3$. (The difference between this case and positronium lies partly in the fact that the gluons must be emitted in a color singlet state.) Consequently, one expects η_c to have a hadronic width of a few MeV.

[7]The second choice of barrier factor also corresponds to the nonrelativistic quark model, as in the calculation of Callan, et al., (1975).

[8]The fact that $\Gamma(K^+ \to \pi^+\pi^0)/\Gamma(K_S^0 \to \pi\pi) \sim 1.5 \times 10^{-3}$ indicates that this suppression may be important. On the other hand, exoticity of final states may have nothing to do with the suppression of $K^+ \to \pi^+\pi^0$; it may simply be a reflection of transformation properties (such as $\Delta I = \frac{1}{2}$) of the interaction.

[9]This mass is not far from that of the ϕ_c'. The two would be degenerate for a Coulomb potential.

[10]See Table 1 of (Moffeit, 1973).

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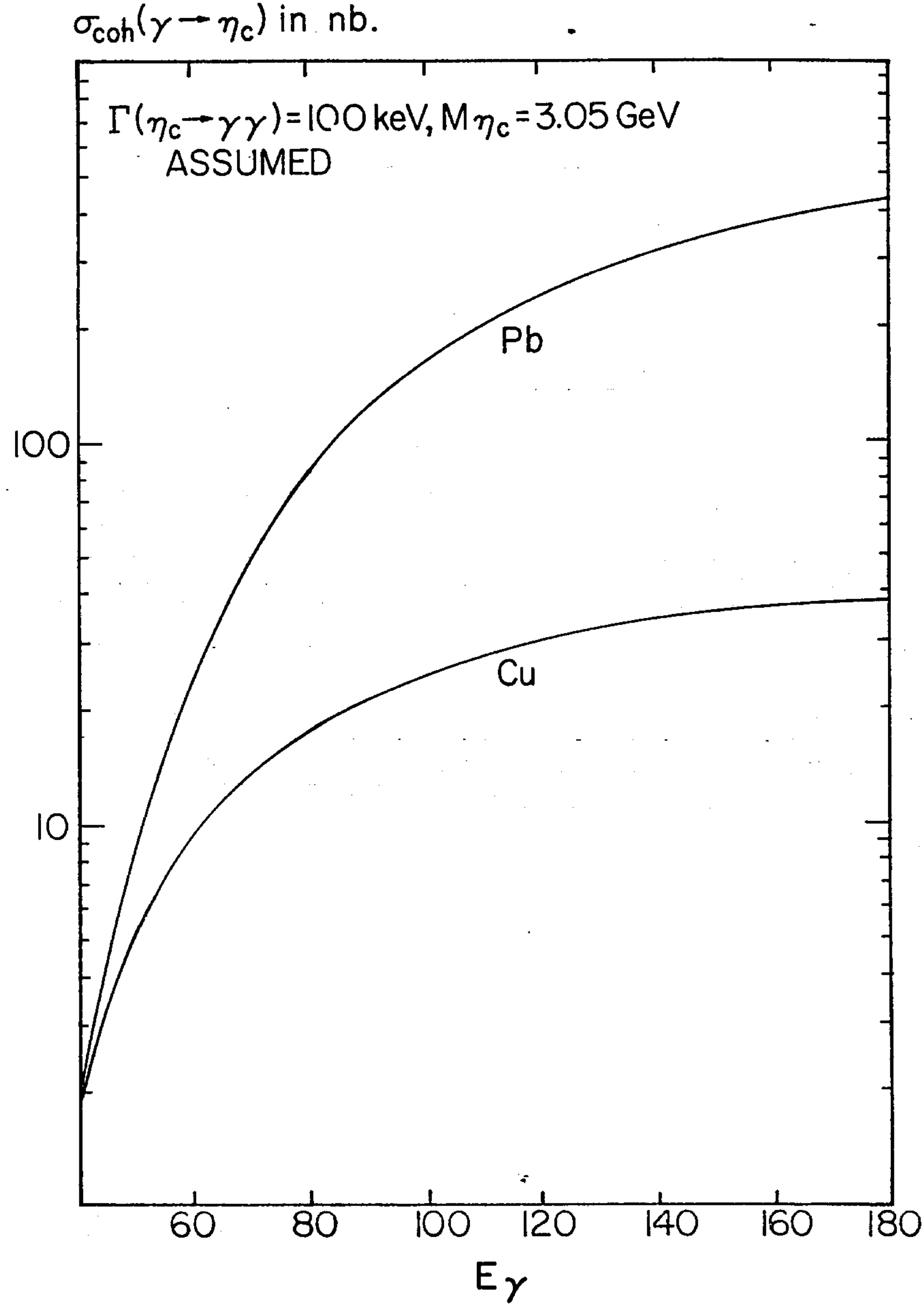
FIGURE CAPTIONS

Fig. A.1 Coherent Primakoff production cross section of η_c as a function of laboratory photon energy on targets Pb and Cu (a); Be and H (b). $\Gamma(\eta_c \rightarrow \gamma\gamma)$ of 100 keV and mass of η_c = 3.05 are assumed. The cross section σ and $\Gamma(\eta_c \rightarrow \gamma\gamma)$ are proportional.

Fig. A.2 Mueller-Regge diagram illustrating production of particle A in the central region.

Fig. A.3 Mass spectrum of the Ke system in the decay $D \rightarrow Ke\nu$. M(D) = 2.2 is assumed. In one case $m_V = \infty$, the form factor is assumed constant. In another $m_V = 2.4$, the form factor is parametrized as $m_V^2/(q^2 - m_V^2)$.



$\sigma_{coh}(\gamma \to \eta_c)$ in nb.
$\Gamma(\eta_c \to \gamma\gamma) = 100$ keV, $M\eta_c = 3.05$ GeV
ASSUMED
Pb
Cu
100
10
60
80
100
120
140
160
180
E_γ

FIG. A1 a

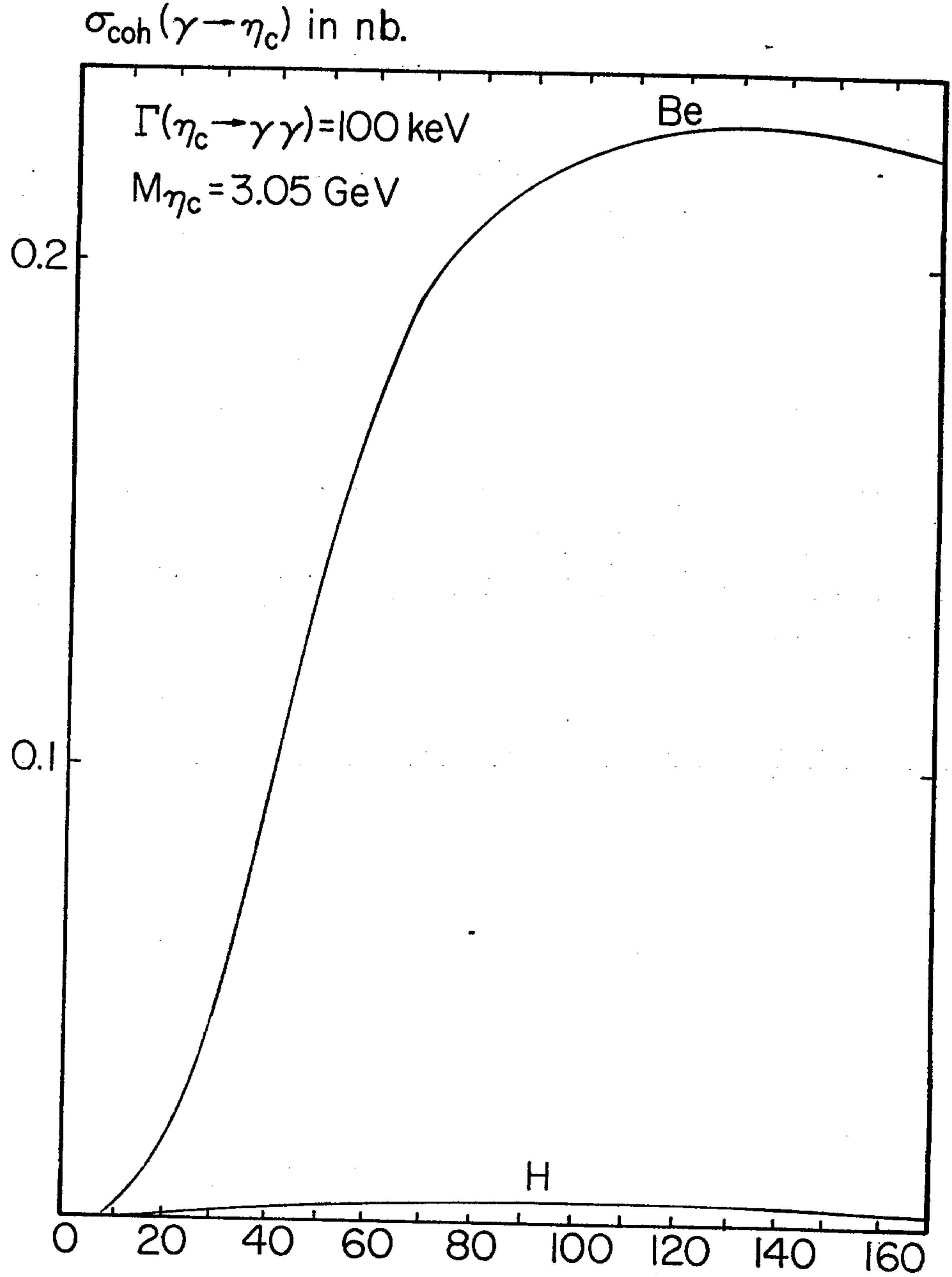

$\sigma_{coh}(\gamma \to \eta_c)$ in nb.
$\Gamma(\eta_c \to \gamma\gamma) = 100$ keV
$M_{\eta_c} = 3.05$ GeV
Be
H
0.2
0.1
0
20
40
60
80
100
120
140
160

FIG. A1b

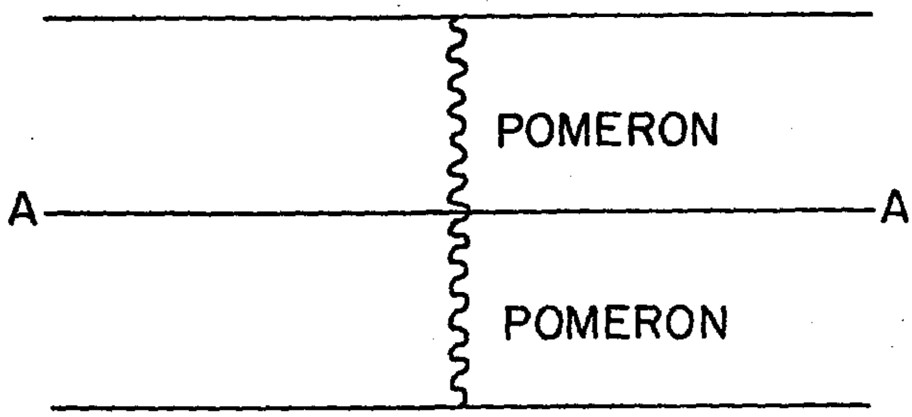


FIG. A2

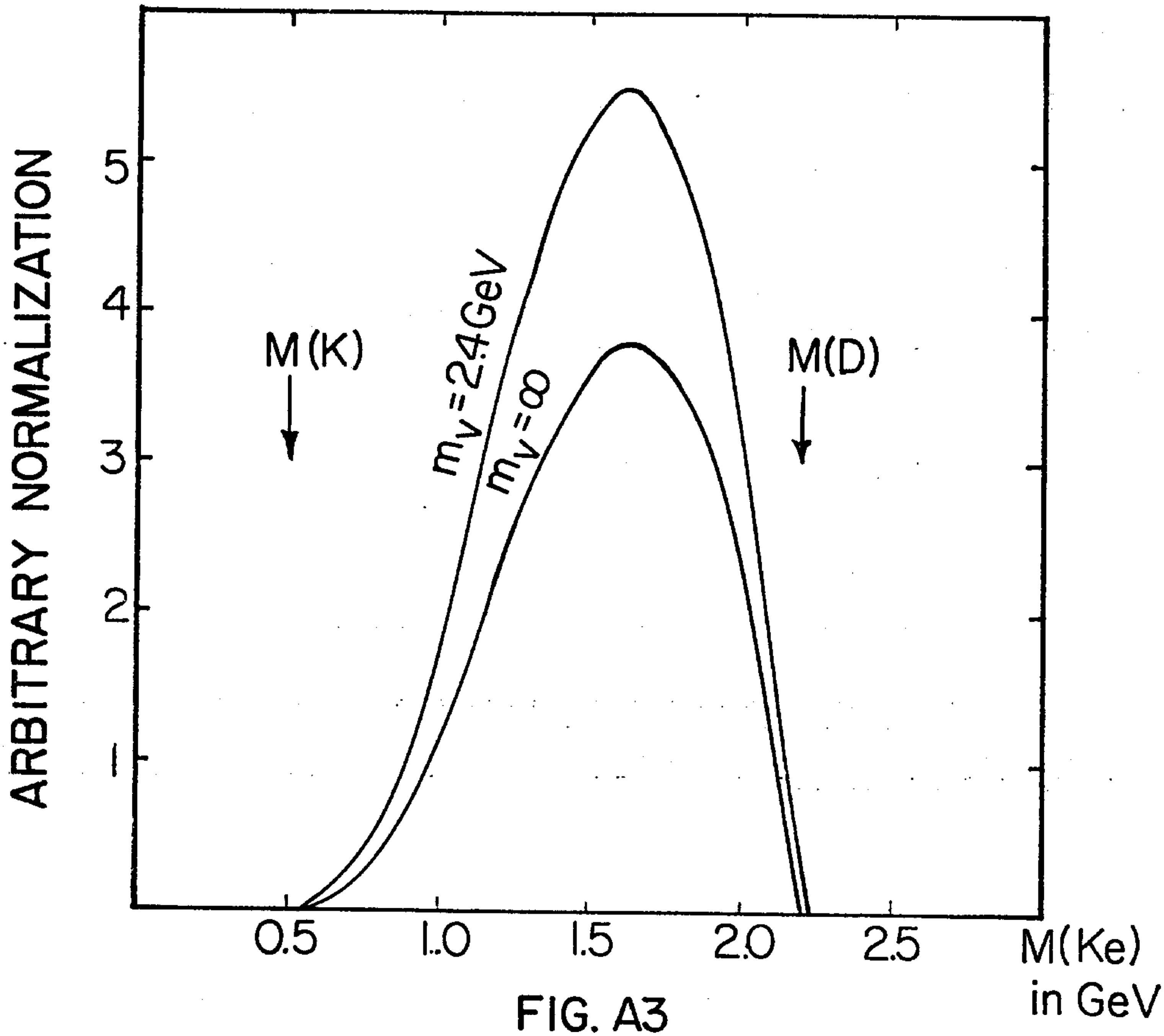

ARBITRARY NORMALIZATION
1
2
3
4
5
M(K)
M(D)
$m_\nu = 24\,GeV$
$m_\nu = \infty$
0.5
1.0
1.5
2.0
2.5
M(Ke) in GeV

FIG. A3